**Approach # 1: Scenario 3: ≠1**

***Note:*** *This section is removed from the sizing paper and shortened for the sake of brevity.*

The objective function is as follows:

|  |  |
| --- | --- |
|  | (1) |

Where:

|  |  |
| --- | --- |
|  | (2) |

Replacing “” as follow

|  |  |
| --- | --- |
|  | (3) |

The new objective will be:

|  |  |
| --- | --- |
|  | (4) |

Where:

|  |  |
| --- | --- |
|  | (5) |

Recopying the following constraints from (Scenario 1: with η ≠1):

|  |  |
| --- | --- |
|  | (6) |
|  | (7) |

The binary variable in the rate () depends on the grid energy

is one if the grid energy is positive and is zero if negative. Let’s assume the following constraint for a second:

|  |  |
| --- | --- |
| Rewrite equation (8): | (8) |
|  | (9) |
|  |  |

|  |  |
| --- | --- |
|  | (10) |

Using Big-M and binary variable to assign value for as following:

|  |  |
| --- | --- |
| *+M* | (11) |
|  | (12) |

That means if the grid energy is positive, is 1 and if negative is 0:

The objective function (eq. (4)) needs to be linearized to be solvable using convex optimization: we assume the following two new variables:

|  |  |
| --- | --- |
|  | (13) |

The constraints on u, given below:

|  |  |
| --- | --- |
|  | (14) |

Applying the two new variables (, ), given in eq. (13), on the constraints of eq. (14):

|  |  |
| --- | --- |
|  | (15) |

Subtracting eq. (15) from eq. (14):

|  |  |
| --- | --- |
| u -  u - | (16) |

Rewriting the objective function:

|  |  |
| --- | --- |
|  | (17) |

The objective function, eq. (17), needs to be further linearized by assuming the following new variables:

|  |  |
| --- | --- |
|  | (18) |

Applying these two new variables on eq. (15)

|  |  |
| --- | --- |
|  | (19) |

Subtracting eq. (19) from eq. (15)

|  |  |
| --- | --- |
|  | (20) |

The new objective function will be:

|  |  |
| --- | --- |
|  | (21) |

Where:

|  |  |
| --- | --- |
|  | (22) |

However, the variables in the constraints of eq. (20) need to be linearized. To replace the multiplication of two binary variables with one variable we assume new binary variable as following (= ) and ( = ) and we use the following constraints:

|  |  |
| --- | --- |
|  | (23) |

The new objective function with all constraints will be as follows:

|  |  |
| --- | --- |
|  | (24) |

Where:

|  |  |
| --- | --- |
|  | (25) |

Variables u, *, ,* , and , *, , , ,* , and

Subject to:

|  |  |
| --- | --- |
| *u -*  *u -*  *+M* | (26) |